Vestnik of Don State Technical University. 2018. Vol. 18, no. 4, pp. 350–361. ISSN 1992-5980 eISSN 1992-6006
Вестник Донского государственного технического университета. 2018. Т. 18, № 4. С. 350–361. ISSN 1992-5980 eISSN 1992-6006

МЕХАНИКА
MECHANICS

УДК:517.95, 519.6 https://doi.org/10.23947/1992-5980-2018-18-4-350-361

Development and correctness analysis of the mathematical model of transport and suspension sedimentation depending on bottom relief variation [*]

A. I. Sukhinov[1], V. V. Sidoryakina[2]**
[1] Don State Technical University, Rostov-on-Don, Russian Federation
[2] Taganrog Chekhov Institute, Rostov State University of Economics (RINH) branch, Taganrog, Russian Federation

Построение и исследование корректности математической модели транспорта и осаждения взвесей с учетом изменения рельефа дна[***]

А. И. Сухинов[1], В. В. Сидорякина[2]**
[1] Донской государственный технический университет, г. Ростов-на-Дону, Российская Федерация
[2] Таганрогский институт имени А. П. Чехова (филиал) РГЭУ (РИНХ), г. Таганрог, Российская Федерация

Introduction. The paper is devoted to the study on the three-dimensional model of transport and suspension sedimentation in the coastal area due to changes in the bottom relief. The model considers the following processes: advective transfer caused by the aquatic medium motion, micro-turbulent diffusion, and gravity sedimentation of suspended particles, as well as the bottom geometry variation caused by the particle settling or bottom sediment rising. The work objective was to conduct an analytical study of the correctness of the initial-boundary value problem corresponding to the constructed model.

Materials and Methods. The change in the bottom relief aids in solution to the initial-boundary value problem for a parabolic equation with the lowest derivatives in a domain whose geometry depends on the desired function of the solution, which in general leads to a nonlinear formulation of the problem. The model is linearized on the time grid due to the "freezing" of the bottom relief within a single step in time and the subsequent recalculation of the bottom surface function on the basis of the changed function of the suspension concentration, as well as a possible change in the velocity vector of the aquatic medium.

Research Results. For the linearized problem, a quadratic functional is constructed, and the uniqueness of the solution to the corresponding initial boundary value problem is proved within the limits of an unspecified time step. On the basis of the quadratic functional transformation, we obtain a prior estimate of the solution norm in the functional space L2 as a function of the integral time estimates of the right side, and the initial condition. Thus, the stability of the solution to the initial

Введение. Настоящая работа посвящена исследованию пространственно-трехмерной модели транспорта и осаждения взвеси в прибрежной зоне с учетом изменения рельефа дна. Модель учитывает следующие процессы: адвективный перенос, обусловленный движением водной среды, микротурбулентную диффузию и гравитационное осаждение частиц взвеси, а также изменение геометрии дна, вызванное осаждением частиц взвеси или подъемом частиц донных отложений.

Целью работы являлось проведение аналитического исследования корректности начально-краевой задачи, соответствующей построенной модели.

Материалы и методы. Изменение рельефа дна приводит к необходимости решать начально-краевую задачу для уравнения параболического типа с младшими производными в области, геометрия которой зависит от искомой функции решения, что приводит, в общем случае, к нелинейной постановке задачи. Выполнена линеаризация модели на временной сетке за счет «замораживания» рельефа дна в пределах одного шага по времени и последующего пересчета функции поверхности дна на основе изменившейся функции концентрации взвешенного вещества, а также возможного изменения вектора скорости движения водной среды.

Результаты исследования. Для линеаризованной задачи построен квадратичный функционал и энергетическим методом доказана единственность решения соответствующей начально-краевой задачи в пределах произвольного шага по времени. На основе преобразования квадратичного функционала получена априорная оценка нормы решения в функциональном пространстве L_2 в зависимости от интегральных оценок по времени правой части, граничных условий и начального условия, и, таким образом, доказана устойчивость решения исходной задачи при из-

[*] The research is done on theme no. 2.6905.2017/БЧ within the frame of the government task of RF Ministry of Education and Science in R&D.
[**] E-mail: sukhinov@gmail.com, cvv9@mail.ru
[***] Работа выполнена по теме № 2.6905.2017/БЧ в рамках госзадания Минобрнауки России в части НИР.

problem from the change of the initial and boundary conditions, the right-hand side function, is established.
Discussion and Conclusions. The model can be of value for predicting the spread of contaminants and changes in the bottom topography, both under an anthropogenic impact and due to the natural processes in the coastal area.

менении начального и граничных условий, функции правой части.
Обсуждение и заключения. Модель может представлять ценность при прогнозе распространения загрязнений и изменения рельефа дна, как при антропогенном воздействии, так и в силу естественно протекающих природных процессов в прибрежной зоне.

Keywords: coastal systems, mathematical model, diffusion-convection problems of suspension sedimentation, bottom relief change, uniqueness of solution, and stability of initial-boundary value problem.

Ключевые слова: прибрежные системы, математическая модель, задачи диффузии-конвекции осаждения взвешенного вещества, изменение рельефа дна, единственность решения и устойчивость начально-краевой задачи.

For citation: A.I. Sukhinov, V.V. Sidoryakina.. Development and correctness analysis of the mathematical model of transport and suspension sedimentation depending on bottom relief variation. Vestnik of DSTU, 2018, vol. 18, no. 4, pp.350−361. https://doi.org/10.23947/1992-5980-2018-18-4-350-361

Образец для цитирования: Сухинов, А. И. Построение и исследование корректности математической модели транспорта и осаждения взвесей с учетом изменения рельефа дна / А. И. Сухинов, В. В. Сидорякина // Вестник Дон. гос. техн. ун-та. — 2018. — Т. 18, № 4. — С.350−361. https://doi.org/10.23947/1992-5980-2018-18-4-350-361

Introduction. The aquatic habitat protection [1−2] is one of the most important factors that determine the integrated research development of the coastal areas. Damage control over the natural processes, such as pollution, sedimentation, and depletion of water areas, leads to necessity for studying all aspects that affect changes in coastal waters. Maintenance of water bodies in proper condition and timely intervention in its operation mode is directly related to the increase in port capacity and the efficient development of the coastal infrastructure (ensuring an accessway to the berths of ships with a low landing; desilting and aquatic vegetation clearing of the coastal strip; etc.) [3-5]. As a rule, research practice in this field requires the construction of mathematical models that are as close as possible to real processes [6−11].

A continuous mathematical model describing spatial-three-dimensional processes associated with transport and gravitational suspension sedimentation in the aquatic medium with varying bottom relief is considered. This model takes into account micro-turbulent diffusion and advective transfer of suspensions, the effect of gravity on suspension, the presence of the bottom and a free surface, and a bottom contour variation.

The suspension transport model enables to study the hydrophysical processes of aquatic systems, to predict the dynamics of the bottom surface change based on the description of the lifting, transport, sedimentation, changes in the concentration of suspension [12–13]. The uniqueness of the solution to the corresponding initial-boundary value problem is proved, and a prior estimate of the solution norm is obtained depending on the integral estimation of the right-hand side, boundary conditions, and the initial condition.

Materials and Methods. Continuous 3D model of suspension diffusion-convection and the corresponding initial boundary value problem. Consider a continuous mathematical model of sediment spreading in the aqueous media considering diffusion and convection of suspension, gravity action on suspension, presence of the bottom and a free surface. We will use $Oxyz$ Cartesian coordinate system where Ox axis passes along the nonperturbed water surface and is directed toward the sea, and Oz axis is directed vertically downwards. Assume that $h=H+\eta$ is the total water depth, m; H is depth with undisturbed water surface, m; η is elevation of the free surface relative to the geoid (sea level), m (Fig. 1).

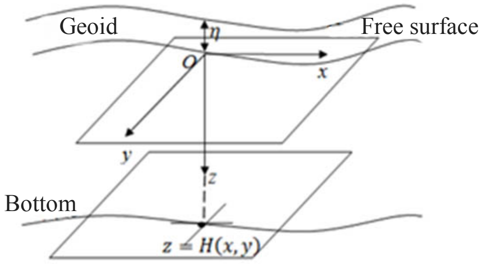

Fig. 1. Introduction of $Oxyz$ coordinate system $Oxyz$

Suppose that in $\bar{G}=\left\{0\le x\le L_x,\, 0\le y\le L_y,\, 0\le z\le H(x,y)\right\}$ closure region, there are suspensions which have $c=c(x,y,z,t)$ concentration at (x,y,z) point and at t time, mg/l; t is temporary variable, sec. We will also use $L_z\equiv\max\limits_{0\le x\le L_x,\,0\le y\le L_y} H(x,y)$ notation.

The behavior of the suspended particles will be described by the following system of equations:

$$\begin{cases}\dfrac{\partial c}{\partial t}+\dfrac{\partial(uc)}{\partial x}+\dfrac{\partial(vc)}{\partial y}+\dfrac{\partial\left((w+w_g)c\right)}{\partial z}=\mu_h\left(\dfrac{\partial^2 c}{\partial x^2}+\dfrac{\partial^2 c}{\partial y^2}\right)+\dfrac{\partial}{\partial z}\left(\mu_v\dfrac{\partial c}{\partial z}\right)+F,\\ \dfrac{\partial H}{\partial t}=-\dfrac{\varepsilon}{\rho}w_g c,\end{cases} \quad (1)$$

where *u, v, w* are components of $\vec{U}$ fluid velocity, m/s; w_g is hydraulic size or sedimentation rate, m/s; μ_h, μ_v are coefficients of the horizontal and vertical turbulent diffusion of particles, respectively, m^2/s; F is power of particle sources; ε is porosity of bottom materials.

Summands on the left side (except for the time derivative) of the first equation of the system (1) describe the advective particle transport due to the inertial motion of the aqueous media, as well as sedimentation under the action of gravity. The summands on the right side describe the suspension diffusion. The vertical diffusion coefficient is chosen different from the horizontal diffusion coefficient due to the fact that the effect of difference between these coefficients is often observed in various media and can be caused by various factors.

As G region, we consider $ABCDA_1OC_1D_1$ "parallelepiped" "skewed" to the shore, whose $A_1OC_1D_1$ upper base lies on $(z=0)$ free surface, and $(z=H(x,y))$ part of the bottom surface is its lower base. Suppose S is $\bar{G}$ surface, $\vec{n}$ is the outward normal to the surface of the "skewed parallelepiped". We assume the given $\overrightarrow{U^*}$ as the fluid velocity on $\bar{G}$ side surfaces. Complete with the boundary conditions of first kind for the particle concentration function, this allows determining the suspension flow both towards the coast and along the coast (Fig. 2).

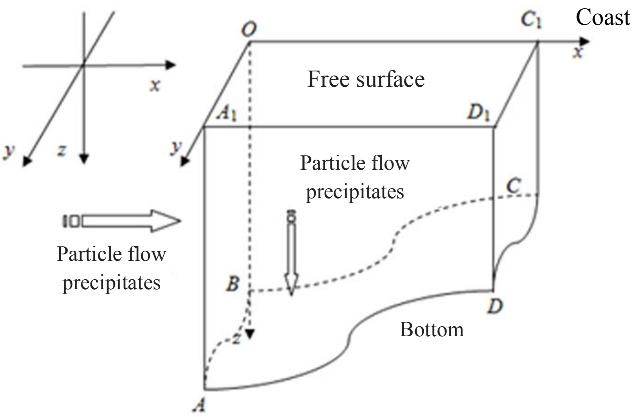

Fig. 2. Solution area for suspension transport

Add the initial and boundary conditions (assuming that the sedimentation is irreversible) to the system (1)

As the initial conditions at $t=0$ time, we accept

$$c(x,y,z,0)\equiv c_0(x,y,z); \quad (2)$$

$$H(x,y,0)=H_0(x,y). \quad (3)$$

We set boundary conditions on $ABCDA_1OC_1D_1$ faces (we set suspended flows both towards the coast and along the coast):

- on the faces $S_1\equiv AA_1OB$ $\left(x=0,\ 0\le y\le L_y,\ 0\le z\le L_z\right)$, $S_2\equiv AA_1D_1D$ $\left(y=L_y,\ 0\le x\le L_x,\ 0\le z\le L_z\right)$ and $S_3\equiv BOC_1C$ $\left(y=0,\ 0\le x\le L_x,\ 0\le z\le L_z\right)$

$$c=c^*,\ \text{где}\ c^*=c^*(x,y,z,t),\ \ t\in[0,T]; \quad (4)$$

- on the faces $S_4\equiv DD_1C_1C$ $\left(x=L_x,\ 0\le y\le L_y,\ 0\le z\le L_z\right)$ and $S_5\equiv A_1OC_1D_1$ $\left(z=0,\ 0\le x\le L_x,\ 0\le y\le L_y\right)$

$$c=0; \quad (5)$$

- on the surface $S_6\equiv ABCD$ $\left(z=H(x,y,t),\ 0\le x\le L_x,\ 0\le y\le L_y\right)$

$$\frac{\partial c}{\partial n} = -\frac{w_g}{\mu_v} c \text{ или } \frac{\partial c}{\partial z} = -\frac{w_g}{\mu_v} c. \quad (6)$$

The boundary condition (5) occurs with a relatively small slope of the bottom:

$$\max_{S_6} \sqrt{\left(\frac{\partial H}{\partial x}\right)^2 + \left(\frac{\partial H}{\partial y}\right)^2} \ll 1.$$

The following condition of the solution domain nondegeneracy is set up for all (x, y, t) at which the initial boundary value problem is formulated:

$$H(x, y, t) \geq h_0 \equiv const > 0, \quad 0 \leq t \leq T. \quad (7)$$

When studying combined models of sediment and suspension transport, it is possible to increase the concentration of suspended particles in the bottom layer due to the rising bottom sediment particles if the shear stress exceeds of a certain critical value is exceeded[13–16]. Then, instead of the boundary condition (6), we will consider the boundary condition of the following form

$$\frac{\partial c}{\partial z} = \alpha c, \quad \alpha = const > 0. \quad (8)$$

Linearization of the initial-boundary value problem of transport and suspension sedimentation. To create a linearized model on $0 \leq t \leq T$ time interval, we construct a uniform grid ωτ with a step τ, that is, a set of points $\omega_\tau = \{t_n = n\tau, \ n = 0,1,...,N, \ N\tau = T\}$.

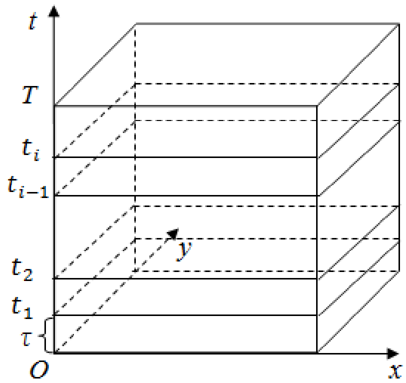

Fig. 3.Construction of time grid

$c^{(n)}(x, y, z, t_{n-1})$ and $H^{(n)}(x, y, t_{n-1})$ functions are determined at each step of ω_τ time grid. If $n = 1$, then functions of the initial condition will suffice to $c^{(1)}(x, y, z, t_0)$, $H^{(1)}(x, y, t_0)$, viz $c^{(1)}(x, y, z, 0) \equiv c_0(x, y, z)$, $H^{(1)}(x, y, t_0) \equiv H_0(x, y)$ respectively. But if $n = 2,..., N$, then $c^{(n)}(x, y, z, t_{n-1}) = c^{(n-1)}(x, y, z, t_{n-1})$ functions are assumed to be known, since the problem (1)–(6) for the previous $t_{n-2} < t \leq t_{n-1}$ time interval is supposed to be solved.

We write the system (1) on $t_{n-1} < t \leq t_n$ interval in the form:

$$\begin{cases} \dfrac{\partial c^{(n)}}{\partial t} + \dfrac{\partial \left(u c^{(n)}\right)}{\partial x} + \dfrac{\partial \left(v c^{(n)}\right)}{\partial y} + \dfrac{\partial \left(\left(w + w_g\right) c^{(n)}\right)}{\partial z} = \mu_h \left(\dfrac{\partial^2 c^{(n)}}{\partial x^2} + \dfrac{\partial^2 c^{(n)}}{\partial y^2}\right) + \dfrac{\partial}{\partial z}\left(\mu_v \dfrac{\partial c^{(n)}}{\partial z}\right) + F, \\ \dfrac{\partial H^{(n)}}{\partial t} = -\dfrac{\varepsilon}{\rho} w_g c^{(n)} \end{cases} \quad (9)$$

and complete it with the initial conditions:

$$c^{(1)}(x, y, z, t_0) = c_0(x, y, z), c^{(n)}(x, y, z, t_{n-1}) = c^{(n-1)}(x, y, z, t_{n-1}), \quad n = 2,..., N. \quad (10)$$

$$H^{(1)}(x, y, t_0) = H_0(x, y), H^{(n)}(x, y, t_{n-1}) = H^{(n-1)}(x, y, t_{n-1}), \quad n = 2,..., N. \quad (11)$$

The boundary conditions (4)–(6) are assumed to be fulfilled for all $t_{n-1} \leq t \leq t_n$ time intervals.

By defining $c^{(n)}\left(x,y,z,t_{n-1}\right)=c^{(n-1)}\left(x,y,z,t_{n-1}\right)$ function on $t_{n-1}<t\le t_n$ time interval, we can find $H^{(n)}\left(x,y,t_{n-1}\right)$ function. For this end, we integrate both members of the second equation of the system (9) over $t_{n-1}\le t\le t_n$ variable. We will get

$$\int_{t_{n-1}}^{t_n}\frac{\partial H^{(n)}}{\partial t}dt=-\frac{\varepsilon}{\rho}w_g\int_{t_{n-1}}^{t_n}c^{(n)}dt. \tag{12}$$

From the equality (12), it is not difficult to get

$$H^{(n)}=H^{(n-1)}-\frac{\varepsilon}{\rho}w_g\sum_{n=1}^{N}\int_{t_{n-1}}^{t_n}c^{(n)}dt. \tag{13}$$

We introduce $G_{n-1}=\left\{0<x<L_x,\,0<y<L_y,\,0<z<H^{n-1}\left(x,y,t_{n-1}\right)\right\}$ domain at each $t_{n-1}\le t\le t_n$ time step.

We have a chain of linear initial-boundary value problems for each time layer, where the system of the type

$$\begin{cases}\dfrac{\partial c^{(n)}}{\partial t}+\dfrac{\partial\left(uc^{(n)}\right)}{\partial x}+\dfrac{\partial\left(vc^{(n)}\right)}{\partial y}+\dfrac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}=\mu_h\left(\dfrac{\partial^2c^{(n)}}{\partial x^2}+\dfrac{\partial^2c^{(n)}}{\partial y^2}\right)+\dfrac{\partial}{\partial z}\left(\mu_v\dfrac{\partial c^{(n)}}{\partial z}\right)+F, & (14)\\ \left(x,y,z\right)\in G_{n-1}\ ,\quad G_{n-1}=\left\{0<x<L_x,\ 0<y<L_y,\ 0<z<H^{n-1}\left(x,y,t_{n-1}\right)\right\}, & \\ H^{(n)}=H^{(n-1)}-\dfrac{\varepsilon}{\rho}w_g\displaystyle\sum_{n=1}^{N}\int_{t_{n-1}}^{t_n}c^{(n)}dt,\quad n=1,2,...,N. & (15)\end{cases}$$

is considered for $t_{n-1}\le t\le t_n$ interval with the initial conditions:

$$c^{(n)}\left(x,y,z,t_{n-1}\right)=c^{(n-1)}\left(x,y,z,t_{n-1}\right), \tag{16}$$

$$H^{(n)}\left(x,y,t_{n-1}\right)=H^{(n-1)}\left(x,y,t_{n-1}\right). \tag{17}$$

Note that at each time step, the boundary surfaces will change (except S_5 face). Considering $t_{n-1}\le t\le t_n$ time interval, we set the boundary conditions on the edges of G_{n-1} domain:

- on $S_{1,n-1}\left(x=0,\ 0\le y\le L_y,\quad 0\le z\le H^{(n-1)}\left(0,y,t_{n-1}\right)\right)$, $S_{2,n-1}\left(y=L_y,\ 0\le x\le L_x,\ 0\le z\le H^{(n-1)}\left(x,L_y,t_{n-1}\right)\right)$ and $S_{3,n-1}\left(y=0,\ 0\le x\le L_x,\ 0\le z\le H^{(n-1)}\left(x,0,t_{n-1}\right)\right)$ faces

$$c^{(n)}=c^*\text{, где }c^*=c^*\left(x,y,z,t\right),\quad t\in\left[t_{n-1},t_n\right]; \tag{18}$$

- on $S_{4,n-1}\left(x=L_x,\ 0\le y\le L_y,\quad 0\le z\le H^{(n-1)}\left(L_x,y,t_{n-1}\right)\right)$ и $S_{5,n-1}\left(z=0,\ 0\le x\le L_x,\quad 0\le y\le L_y\right)\equiv A_1OC_1D_1$ faces

$$c^{(n)}=0\,; \tag{19}$$

- on $S_{6,n-1}\left(z=H^{(n-1)}\left(x,y,t_{n-1}\right),\ 0\le x\le L_x,\quad 0\le y\le L_y\right)$ surface

$$\frac{\partial c^{(n)}}{\partial n}=-\frac{w_g}{\mu_v}c^{(n)}\text{ или }\frac{\partial c^{(n)}}{\partial z}=-\frac{w_g}{\mu_v}c^{(n)}. \tag{20}$$

The boundary condition (8) will be replaced by the following

$$\frac{\partial c^{(n)}}{\partial z}=\alpha c^{(n)},\quad\alpha=const>0. \tag{21}$$

Thus, it is supposed that the bottom relief within this time step, when calculating the distribution of suspension concentrations, does not change and is taken from the previous time layer. First of all, at this $t_{n-1}\le t\le t_n$ time step, the initial-boundary value problem for the convection-diffusion equation (14) with $H^{(n-1)}$ fixed bottom relief function is solved, and only then the update (recomputation) of $H^{(n)}$ relief function is performed in accordance with the equality (15).

The determination of the conditions of existence, uniqueness and continuous dependence of the solution on the input problem data is carried out on a fixed time layer under these assumptions and subject to the condition (7).

The authors do not plan to study the existence of solutions to the initial-boundary value problems (14) - (20) and (14) - (19), (21) in this paper. Questions of the existence of solutions to the initial-boundary value problems for parabolic equations with lower derivatives (diffusion-convection equations) are considered, for example, in the monographs [17–18].

Research Results. Investigating uniqueness of the solution to the initial-boundary problem of suspension transport.

Consider the initial boundary value problem (14) - (20) formulated for the arbitrary $t_{n-1} < t \le t_n$ time layer.

Multiply the left and right member of equation (14) by $c^{(n)}$ function and get:

$$\frac{\partial c^{(n)}}{\partial t}+\frac{\partial\left(uc^{(n)}\right)}{\partial x}+\frac{\partial\left(vc^{(n)}\right)}{\partial y}+\frac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}=\mu_h c^{(n)}\left(\frac{\partial^2 c^{(n)}}{\partial x^2}+\frac{\partial^2 c^{(n)}}{\partial y^2}\right)+c^{(n)}\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)+c^{(n)}F\,. \tag{22}$$

The left member of the equality (22) can be transformed as follows:

$$c^{(n)}\frac{\partial c^{(n)}}{\partial t}+c^{(n)}\left(\frac{\partial\left(uc^{(n)}\right)}{\partial x}+\frac{\partial\left(vc^{(n)}\right)}{\partial y}+\frac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}\right)=\frac{1}{2}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}+c^{(n)}div\left(c^{(n)}\vec{U}\right)=$$
$$=\frac{1}{2}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}+\frac{1}{2}div\left(\left(c^{(n)}\right)^2\vec{U}\right)\quad, \tag{23}$$

where $\vec{U}=\left\|u,\,v,\,w+w_g\right\|^T$.

With regard to (23), the equation (22) will be written as

$$\frac{1}{2}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}+\frac{1}{2}div\left(\left(c^{(n)}\right)^2\vec{U}\right)=\mu_h c^{(n)}\left(\frac{\partial^2 c^{(n)}}{\partial x^2}+\frac{\partial^2 c^{(n)}}{\partial y^2}\right)+c^{(n)}\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)+c^{(n)}F\,. \tag{24}$$

Then we integrate both members of the equation (24) over $t_{n-1} \le t \le t_n$ interval, and, after that, over the spatial variables in G_{n-1} domain. In the first term, the order of integration is changed due to the Fubini theorem [19]. We obtain

$$\iiint\limits_{G_{n-1}}\frac{1}{2}\left(\int\limits_{t_{n-1}}^{t_n}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}dt\right)dG_{n-1}+\int\limits_{t_{n-1}}^{t_n}\frac{1}{2}\left(\iiint\limits_{G_{n-1}}div\left(\left(c^{(n)}\right)^2\vec{U}\right)dG_{n-1}\right)dt=$$
$$=\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}\mu_h\left(\frac{\partial^2 c^{(n)}}{\partial x^2}+\frac{\partial^2 c^{(n)}}{\partial y^2}\right)dG_{n-1}\right)dt+\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)dG_{n-1}\right)dt+ \tag{25}$$
$$\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}FdG_{n-1}\right)dt.$$

The first term on the left side of the equation (25) is obviously equal to

$$\iiint\limits_{G_{n-1}}\frac{1}{2}\left(\int\limits_{t_{n-1}}^{t_n}\frac{\partial\left(c^{(n)}\right)^2}{\partial t}dt\right)dG_{n-1}=\iiint\limits_{G_{n-1}}\frac{1}{2}\left(\left(c^{(n)}\right)^2\left(x,y,z,t_n\right)-\left(c^{(n)}\right)^2\left(x,y,z,t_{n-1}\right)\right)dG_{n-1}\,. \tag{26}$$

Next, we turn to the transformation of the second term of the left-hand side of the equality (25). Considering the Gauss-Ostrogradsky formula and the boundary conditions (18) - (20), it can be written as [20]:

$$\int\limits_{t_{n-1}}^{t_n}\left(\frac{1}{2}\iiint\limits_{G_{n-1}}div\left(\left(c^{(n)}\right)^2\vec{U}\right)dG_{n-1}\right)dt=\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{1,n-1}}\left(c^*\right)^2\left(\overrightarrow{U^*},\vec{n}\right)dydz\right)dt+\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}c^2w_g dxdy\right)dt+$$
$$+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{3,n-1}}\left(c^*\right)^2\left(\overrightarrow{U^*},\vec{n}\right)dxdz\right)dt+\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{2,n-1}}\left(c^*\right)^2\left(\overrightarrow{U^*},\vec{n}\right)dxdz\right)dt=-\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{1,n-1}}\left(c^*\right)^2udydz\right)dt- \tag{27}$$
$$-\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{3,n-1}}\left(c^*\right)^2vdxdz\right)dt+\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{2,n-1}}\left(c^*\right)^2vdxdz\right)dt+\frac{1}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}\left(c^{(n)}\right)^2w_g dxdy\right)dt.$$

where $\overrightarrow{U^*}$ is the known velocity of the aquatic medium on the faces where the boundary conditions of the first kind are specified; in fact, these are all side faces, except for $S_{4,n-1}$ and $S_{5,n-1}$ top cover on which the suspension concentration is zero, and therefore the flows through them are zero.

Let us turn to the transformation of the right side of the equation (25). The following equality occurs

$$\iiint\limits_{G_{n-1}}\left[c^{(n)}\left(\mu_h\frac{\partial}{\partial x}\left(\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right)\right]dG_{n-1}=$$
$$=\iiint\limits_{G_{n-1}}\left[\mu_h\frac{\partial}{\partial x}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right]dG_{n-1}- \quad (28)$$
$$-\iiint\limits_{G_{n-1}}\left[\mu_h\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right]dG_{n-1}.$$

Suppose $\vec{Q}=\{Q_x,Q_y,Q_z\}=\left\{\mu_h c^{(n)}\frac{\partial c^{(n)}}{\partial x},\mu_h c^{(n)}\frac{\partial c^{(n)}}{\partial y},c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right\}$. Then, in virtue of the Gauss-Ostrogradsky theorem, we have:

$$\iiint\limits_{G_{n-1}}\left[\mu_h\frac{\partial}{\partial x}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right]dG_{n-1}=\iiint\limits_{G_{n-1}}div\vec{Q}dG=$$
$$=\iint\limits_{S_{2,n-1}}Q_y dxdz+\iint\limits_{S_{4,n-1}}Q_x dydz+\iint\limits_{S_{3,n-1}}Q_y dxdz+\iint\limits_{S_{1,n-1}}Q_x dydz+\iint\limits_{S_{6,n-1}}Q_z dxdy+\iint\limits_{S_{5,n-1}}Q_z dxdy= \quad (29)$$
$$=\iint\limits_{S_{2,n-1}}Q_y dxdz+\iint\limits_{S_{3,n-1}}Q_y dxdz+\iint\limits_{S_{1,n-1}}Q_x dydz+\iint\limits_{S_{6,n-1}}Q_z dxdy.$$

Transforming each term from the right-hand side of (29) subject to the conditions on the boundary (18) - (20), we obtain

$$\iiint\limits_{G_{n-1}}\left[\mu_h\frac{\partial}{\partial x}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial x}\right)+\mu_h\frac{\partial}{\partial y}\left(c^{(n)}\frac{\partial c^{(n)}}{\partial y}\right)+\frac{\partial}{\partial z}\left(c^{(n)}\mu_v\frac{\partial c^{(n)}}{\partial z}\right)\right]dG_{n-1}= \quad (30)$$
$$=\iint\limits_{S_{2,n-1}}c^*\mu_h\frac{\partial c^*}{\partial y}dxdz+\iint\limits_{S_{3,n-1\ 1}}c^*\mu_h\frac{\partial c^*}{\partial y}dxdz+\iint\limits_{S_{1,n-1}}c^*\mu_h\frac{\partial c^*}{\partial x}dydz-\iint\limits_{S_{6,n-1}}w_g\left(c^{(n)}\right)^2dxdy.$$

In virtue of (26), (28), (29) and (30), the equality (25) takes on form

$$\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_n)dG_{n-1}-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{1,n-1}}\left(\frac{1}{2}\left(c^*\right)^2u+c^*\mu_h\frac{\partial c^*}{\partial x}\right)dydz\right)dt-$$
$$-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{3,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v+c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{2,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v-c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+$$
$$+\frac{3}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}w_g\left(c^{(n)}\right)^2dxdy\right)dt+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left(\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right)dG_{n-1}\right]dt= \quad (31)$$
$$=\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_{n-1})dG_{n-1}+\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}FdG_{n-1}\right)dt.$$

The identity (31) will be fundamental under studying the uniqueness and obtaining a prior estimate of the solution norm of the initial boundary value problem (14) - (20). In case of replacing the boundary condition (20) with the boundary condition (21), the quadratic functional (31) changes as follows:

$$\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_n)dG_{n-1}-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{1,n-1}}\left(\frac{1}{2}\left(c^*\right)^2u+c^*\mu_h\frac{\partial c^*}{\partial x}\right)dydz\right)dt-$$
$$-\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{3,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v+c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{2,n-1}}\left(\frac{1}{2}\left(c^*\right)^2v-c^*\mu_h\frac{\partial c^*}{\partial y}\right)dxdz\right)dt+$$
$$+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}\left(\frac{1}{2}w_g-\alpha\mu_v\right)\left(c^{(n)}\right)^2dxdy\right)dt+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left(\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right)dG_{n-1}\right]dt= \quad (32)$$
$$=\frac{1}{2}\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2(x,y,z,t_{n-1})dG_{n-1}+\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}c^{(n)}FdG_{n-1}\right)dt.$$

Suppose that the equation (14) with the same conditions (16) - (20) satisfy two different solutions to $c_1=c_1(x,y,z,t)$, $c_2=c_2(x,y,z,t)$ problem. For their $\tilde{c}=c_1-c_2$ difference, the following initial-boundary problem is valid:

$$\frac{\partial \tilde{c}}{\partial t}+\frac{\partial\left(u\tilde{c}\right)}{\partial x}+\frac{\partial\left(v\tilde{c}\right)}{\partial y}+\frac{\partial\left(\left(w+w_g\right)\tilde{c}\right)}{\partial z}=\mu_h\left(\frac{\partial^2\tilde{c}}{\partial x^2}+\frac{\partial^2\tilde{c}}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(\mu_v\frac{\partial\tilde{c}}{\partial z}\right), \tag{33}$$

$$\tilde{c}\left(x,y,z,0\right)=0,\ \left(x,y,z\right)\in\overline{G}_{n-1}, \tag{34}$$

- on $S_{1,n-1}$, $S_{2,n-1}$, $S_{3,n-1}$, $S_{4,n-1}$, $S_{5,n-1}$ faces

$$\tilde{c}=c^*-c^*=0; \tag{35}$$

- on $S_{6,n-1}$ surface

$$\frac{\partial\tilde{c}}{\partial z}=-\frac{w_g}{\mu_v}\left(c_1-c_2\right)=-\frac{w_g}{\mu_v}\tilde{c}. \tag{36}$$

For $\tilde{c}$ function, the equality (33) will take the form considering the equalities (34)−(36)

$$\begin{gathered}\frac{1}{2}\iiint\limits_{G_{n-1}}\tilde{c}^2\left(x,y,z,t_n\right)dG_{n-1}+\frac{3}{2}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}w_g\tilde{c}^2dxdy\right)dt+\\+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left[\mu_h\left(\frac{\partial\tilde{c}}{\partial x}\right)^2+\mu_h\left(\frac{\partial\tilde{c}}{\partial y}\right)^2+\mu_v\left(\frac{\partial\tilde{c}}{\partial z}\right)^2\right]dG_{n-1}\right]dt=0.\end{gathered} \tag{37}$$

Since $w_g>0$ and other known values under the sign of integrals are positive μh> 0, μv> 0, then the equality (36) is satisfied only under the condition

$$\tilde{c}\left(x,y,z,t\right)\equiv 0,\quad\left(x,y,z\right)\in G_{n-1},\quad t_{n-1}<t\le t_n, \tag{38}$$

which completes the proof of the uniqueness of the initial-boundary value problem (14) - (20) solution.

In case of replacing the boundary condition (20) by the relation (21), instead of the expression (37), we obtain the following equality

$$\begin{gathered}\frac{1}{2}\iiint\limits_{G_{n-1}}\tilde{c}^2\left(x,y,z,t_n\right)dG_{n-1}+\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{6,n-1}}\left(\frac{1}{2}w_g-\alpha\mu_v\right)\tilde{c}^2dxdy\right)dt+\\+\int\limits_{t_{n-1}}^{t_n}\left[\iiint\limits_{G_{n-1}}\left[\mu_h\left(\frac{\partial\tilde{c}}{\partial x}\right)^2+\mu_h\left(\frac{\partial\tilde{c}}{\partial y}\right)^2+\mu_v\left(\frac{\partial\tilde{c}}{\partial z}\right)^2\right]dG_{n-1}\right]dt=0.\end{gathered} \tag{39}$$

We require the fulfillment of the inequality

$$\frac{1}{2}w_g-\alpha\mu_v\ge 0,\quad\left(x,y,z\right)\in S_{6,n-1},\quad t_{n-1}<t\le t_n$$

or

$$\alpha\le\frac{w_g}{2\mu_v},\quad\left(x,y,z\right)\in S_{6,n-1},\quad t_{n-1}<t\le t_n, \tag{40}$$

then all the terms in the equation (39) are nonnegative, and zero equality is possible if and only if $\tilde{c}\left(x,y,z,t\right)\equiv 0,\quad\left(x,y,z\right)\in G_{n-1},\quad t_{n-1}<t\le t_n$, that means the solution uniqueness and in this case.

Reasoning is similarly repeated for all layers of ω_τ time grid. The modification of the boundary conditions associated with the continuous change in the bottom relief depending on the time variable requires additional study and is going beyond the scope of this article.

Theorem. Suppose we are given a system of equations

$$\begin{cases}\frac{\partial c^{(n)}}{\partial t}+\frac{\partial\left(uc^{(n)}\right)}{\partial x}+\frac{\partial\left(vc^{(n)}\right)}{\partial y}+\frac{\partial\left(\left(w+w_g\right)c^{(n)}\right)}{\partial z}=\mu_h\left(\frac{\partial^2c^{(n)}}{\partial x^2}+\frac{\partial^2c^{(n)}}{\partial y^2}\right)+\frac{\partial}{\partial z}\left(\mu_v\frac{\partial c^{(n)}}{\partial z}\right)+F,\\ \left(x,y,z\right)\in G_{n-1},\quad G_{n-1}=\left\{0<x<L_x,0<y<L_y,0<z<H^{n-1}\left(x,y,t_{n-1}\right)\right\},\\ H^{(n)}=H^{(n-1)}-\frac{\varepsilon}{\rho}w_g\sum\limits_{n=1}^{N}\int\limits_{t_{n-1}}^{t_n}c^{(n)}dt,\quad n=1,2,...,N\end{cases}$$

in $\Omega_{n-1}=G_{n-1}\times\left(t_{n-1}<t<t_n\right),\ G_{n-1}=\left(0<x<L_x,0<y<L_y,0<z<H^{(n-1)}\left(x,y,t_{n-1}\right)\right),$ simply connected domain with a sufficiently smooth boundary defined by the smoothness of $z=H^{(n-1)}\left(x,y\right),\quad 0\le x\le L_x,\quad 0\le y\le L_y$ function with the

initial and boundary conditions (16) - (20). Let the functions of $c^{(n)}\left(x,y,z,t_{n-1}\right)$ solution, the velocity vector of $\left\|u,v,w+w_g\right\|^T$ aquatic medium, $c^{(n-1)}\left(x,y,z,t_{n-1}\right)$ initial condition, $F\left(x,y,z,t\right)$, right member of $c^*\left(x,y,z,t\right)$ boundary condition, $\mu_v=\mu_v\left(z\right),\left(x,y,z\right)\in G_{n-1}$ coefficient of $\mu_v=\mu_v\left(z\right),\left(x,y,z\right)\in G_{n-1}$ vertical turbulent exchange satisfy the following smoothness conditions:

$c^{(n)}\left(x,y,z,t_{n-1}\right)\in C^2\left(\Omega_{n-1}\right)\cap C\left(\bar{\Omega}_{n-1}\right)$, $grad\, c^{(n)}\in C\left(\bar{\Omega}_{n-1}\right)$, $\left\|u,v,w+w_g\right\|^T\in C^1\left(\Omega_{n-1}\right)\cap C\left(\bar{\Omega}_{n-1}\right)$, $c^{(n-1)}\left(x,y,z,t_{n-1}\right)\in C\left(\bar{G}_{n-1}\right)$, $F\left(x,y,z,t\right)\in C\left(\Omega_{n-1}\right)$, $\mu_v\left(x,y,z\right)\in C^1\left(G_{n-1}\right)\cap C\left(\bar{G}_{n-1}\right)$, $c^*\left(x,y,z,t\right)\in C\left(S_{n-1}\right)\times\left[t_{n-1}\le t\le t_n\right]$, $S_{n-1}=\bar{G}_{n-1}\setminus G_{n-1}$, $\dfrac{\partial c^*}{\partial n}\in C\left(\left(0\le x\le L_x,\quad 0\le y\le L_y,\quad z=H^{(n-1)}\left(x,y\right)\right)\times\left[t_{n-1}\le t\le t_n\right]\right)$, as well as $c^*\left(x,y,z,0\right)=c_0\left(x,y,z\right)$, $\left(x,y,z\right)\in S_{n-1}\setminus\left(0<x<L_x,0<y<L_y,\quad z=H^{(n-1)}\left(x,y\right)\right)$, $\dfrac{\partial c_0}{\partial z}=-\dfrac{\mu_v}{w_g}c^*$, $\left(0<x<L_x,0<y<L_y,\quad z=H^{(n-1)}\left(x,y\right)\right)$, conditions of consistency of the boundary and initial conditions, then the solution to this problem exists and is unique.

Comment. In case of replacing the boundary condition (20) with the boundary condition (21), the inequality (40) should be added as a sufficient condition for the previous theorem.

Studying the continuous dependence of the solutions to the initial-boundary value problem of suspension transport on the initial, boundary conditions and the right-hand side function. The next stage is connected with the study of the continuous solution dependence on the functions of the right-hand side, boundary and initial conditions for the system (14)–(15).

Suppose that

$$\begin{gathered} c^*\ge c_0^*\equiv const>0,\\ 0\le x\le L_x,\ 0\le y\le L_y,\ 0<z<H^{(n)}\left(x,y,t_{n-1}\right),\ t_{n-1}\le t\le t_n. \end{gathered} \tag{41}$$

For convenience, we introduce the notations: union of all parts of the lateral cylindrical surface (boundaries of G_{n-1} region) is denoted as $S_{c,n-1}$, and the lower base of region – as G_{n-1} – $S_{b,n-1}$. In virtue of the smoothness conditions listed under the above theorem, extrema of functions on the bounded closed sets are reached:

$$\begin{gathered} M_{1,n-1}\equiv\max_{\Omega_{n-1}}\left\{\left|c^{(n)}\right|\right\},\quad M_{2,n-1}\equiv\max_{S_{n-1}}\left\{\left|\frac{\partial c^{(n)}}{\partial x}\right|,\left|\frac{\partial c^{(n)}}{\partial y}\right|\right\},\\ M_{3,n-1}\equiv\max_{S_{c,n-1}}\left\{\mu_h\right\},\quad M_{4,n-1}\equiv\max_{S_{c,n-1}\times\left[t_{n-1}\le t\le t_n\right]}\left\{\left|u\right|,\left|v\right|\right\},\quad M_{5,n-1}\equiv\min_{\bar{G}_{n-1}}\left\{\mu_h,\mu_v\right\}. \end{gathered} \tag{42}$$

We will focus on the equation (31) if the boundary condition (20) is used, and on the equality (32) in case of the boundary condition (21). Evoking Friedrichs inequality, we have a chain of inequalities:

$$\begin{gathered} \iiint_{G_{n-1}}\left(\mu_h\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\mu_h\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\mu_v\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right)dG_{n-1}\ge\\ \ge\min_{G_{n-1}}\left\{\mu_h,\mu_v\right\}\iiint_{G_{n-1}}\left(\left(\frac{\partial c^{(n)}}{\partial x}\right)^2+\left(\frac{\partial c^{(n)}}{\partial y}\right)^2+\left(\frac{\partial c^{(n)}}{\partial z}\right)^2\right)dG_{n-1}\ge\\ \ge M_{5,n-1}\left(\pi^2\left(\frac{1}{L_x^{\,2}}+\frac{1}{L_y^{\,2}}+\frac{1}{\left(H^{(n-1)}\right)^2}\right)\right)\iiint_{G_{n-1}}\left(c^{(n)}\right)^2dG_{n-1}. \end{gathered} \tag{43}$$

We turn to the equation (26) from which, in virtue of (42) and (43), we obtain the inequality:

$$\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}+2M_{5,n-1}\left(\pi^2\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}+\frac{1}{\left(H^{(n-1)}\right)^2}\right)\right)\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}\right)dt+$$
$$+3\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{b,n-1}} w_g\left(c^{(n)}\right)^2 dxdy\right)dt\le\iiint\limits_{G_{n-1}}c_0^{\,2}dG_{n-1}+M_{4,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left(c^*\right)^2 dS_{n-1}\right)dt+ \quad (44)$$
$$+2M_{2,n-1}M_{3,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left|c^*\right|dS_{n-1}\right)dt+2M_{1,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left|F\right|dG_{n-1}\right)dt.$$

From inequality (44), there are two inequalities

$$\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}\le\iiint\limits_{G_{n-1}}c_0^{\,2}dG_{n-1}+M_{4,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left(c^*\right)^2 dS_{n-1}\right)dt+$$
$$+2M_{2,n-1}M_{3,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left|c^*\right|dS_{n-1}\right)dt+2M_1\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left|F\right|dG_{n-1}\right)dt. \quad (45)$$

and

$$\iiint\limits_{G_{n-1}}\left(c^{(n)}\right)^2 dG_{n-1}\le M_{6,n-1}\left(\iiint\limits_{G_{n-1}}c_0^{\,2}dG_{n-1}+\right.$$
$$\left.+M_{4,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left(c^*\right)^2 dS_{n-1}\right)dt+2M_{2,n-1}M_{3,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iint\limits_{S_{c,n-1}}\left|c^*\right|dS_{n-1}\right)dt+2M_{1,n-1}\int\limits_{t_{n-1}}^{t_n}\left(\iiint\limits_{G_{n-1}}\left|F\right|dG_{n-1}\right)dt\right). \quad (46)$$

where $M_{6,n-1}=\dfrac{1}{2M_{5,n-1}}\left(\pi^2\left(\dfrac{1}{L_x^2}+\dfrac{1}{L_y^2}+\dfrac{1}{\left(H^{(n-1)}\right)^2}\right)\right)^{-1}$.

The inequalities obtained imply the continuous dependence (stability) of the solution to the problem (14) - (20) on the functions of the initial condition, the boundary conditions and the right-hand side, in L_2 norm for any instant of $0<T<+\infty$ time, and also in L_2 time-integral norm.

Obviously, if the inequality (45) and the theorem condition are satisfied, the initial-boundary problem (14) - (19), (20) will also have a solution that depends continuously on the functions of the initial condition, the boundary conditions and the right-hand side in the corresponding norms.

Discussion and Conclusions. Novelty of the proposed non-stationary spatial-three-dimensional mathematical model of suspension transport lies in the fact that, alongside with considering the processes of advective transfer, micro-turbulent diffusion and gravity sedimentation of suspended particles, the model describes the change in bottom geometry caused by the particle settling or bottom sediment rising.

The linearization of the corresponding initial-boundary problem on the time grid is carried out, and the conditions for the uniqueness of the solution to the initial-boundary problem and continuous dependence on the input data – on the functions of the initial condition, boundary conditions, and the right-hand side in L_2 Hilbert space norm in L_2 time integral norm for two variants of boundary conditions are obtained for the arbitrary $t_{n-1}\le t\le t_n$ time step.

References

1. Leontyev, I.O., et al. Pribrezhnaya dinamika: volny, techeniya, potoki nanosov. [Coastal dynamics: waves, flows, deposits drifts.] I.O. Leontyev, ed. Moscow: GEOS, 2001, 272 p. (in Russian).

2. Matishov, G.G., et al. Prirodnye katastrofy v Azovo-Chernomorskom basseyne v nachale XXI veka. [Natural disasters in the Azov-Black Sea basin at the beginning of XXI century.] Rostov-on-Don: SRC RAS Publ. House, 2017, 160 p. (in Russian).

3. Petrov, P.G. Dvizhenie sypuchey sredy v pridonnom sloe zhidkosti. [Motion of a bed load.] Journal of Applied Mechanics and Technical Physics, 1991, vol. 32, iss. 5, pp. 72–76 (in Russian).

4. Barnard, P.L., Jaffe, B.E. and Schoellhamer, D.H. A multi-discipline approach for understanding sediment transport and geomorphic evolution in an estuarine-coastal system—San Francisco Bay. Marine Geology, 2013, vol. 345, pp. 1–2. DOI:10.1016/j.margeo.2013.09.010. DOI: https://doi.org/10.1016/j.margeo.2013.09.010

5. Xiaoying, L., Shi, Q., Yuan, H., Yuehong, C., Pengfei, D. Predictive modeling in sediment transportation across multiple spatial scales in the Jialing River Basin of China. International Journal of Sediment Research, 2015, vol. 30, iss. 3, pp. 250–255.

6. Lusher, A.L., McHugh, M., Thompson, R.C. Occurrence of microplastics in gastrointestinal tract of pelagic and demersal fish from the English channel. Marine Pollution Bulletin, 2013, vol. 67, pp. 94-99.

7. Marchuk, G.I., et al. Matematicheskie modeli v geofizicheskoy gidrodinamike i chislennye metody ikh realizatsii. [Mathematical models in geophysical hydrodynamics and numerical methods for their implementation.] Leningrad: Gidrometeoizdat, 1987, 296 p. (in Russian).

8. Belikov, V.V., Borisova, N.M., Gladkov, G.L. Matematicheskaya model' transporta nanosov dlya rascheta zanosimosti dnouglubitel'nykh prorezey i ruslovykh kar'yerov. [Mathematical model of sediment transport for calculating the sediment accumulation in dredge cuts and channel pits.] Journal of University of Water Communications, 2010, vol. 2, pp. 105–113 (in Russian).

9. Sanne, L.N. Modelling of sand dunes in steady and tidal flow. Denmark: Technical University of Copenhagen, 2003, 185 p.

10. Ballent, A., Pando, S., Purser, A., Juliano, M., Thomsen, L. Modelled transport of benthic marine microplastic pollution in the Nazaré Canyon. Biogeosciences, 2013, vol. 10, pp. 7957-7970. https://doi.org/10.5194/bg-10-7957-2013, 2013. https://doi.org/10.5194/bg-10-7957-2013

11. Miles, J. Wave shape effects on sediment transport. J. Coastal Res., 2013, vol. 2, iss. 65, pp. 1803–1808. DOI::10.2112/SI65-305.1 DOI: https://doi.org/10.2112/SI65-305.1

12. Popkov, V.I. Strukturnye osobennosti i genezis dislokatsiy dna Azovskogo morya. [Structural features and genesis of dislocations of the Sea of Azov bottom.] Geology, Geography and Global Energy, 2008, no. 1, pp. 77–90 (in Russian).

13. Sidoryakina, V.V., Sukhinov, A.I. Issledovanie korrektnosti i chislennaya realizatsiya linearizovannoy dvumernoy zadachi transporta nanosov. [Well-posedness analysis and numerical implementation of a linearized two-dimensional bottom sediment transport problem.] Computational Mathematics and Mathematical Physics, 2017, vol. 57, no. 6, pp. 985–1002 DOI: https://doi.org/10.7868/S0044466917060138 (in Russian). DOI: https://doi.org/10.7868/S0044466917060138

14. Sukhinov, A.I., Sidoryakina, V.V. O skhodimosti resheniya linearizovannoy posledovatel'nosti zadach k resheniyu nelineynoy zadachi transporta nanosov. [Convergence of linearized sequence tasks to the nonlinear sediment transport task solution.] Mathematical Models and Computer Simulations, 2017, vol. 29, no. 11, pp. 19–39 (in Russian). http://mi.mathnet.ru/mm3905

15. Sukhinov, A.I., Sidoryakina, V.V., Sukhinov, A.A. Dostatochnye usloviya skhodimosti polozhitel'nykh resheniy linearizovannoy dvumernoy zadachi transporta nanosov. [Sufficient conditions for convergence of positive solutions to linearized two-dimensional sediment transport problem.] Vestnik of DSTU, 2017, vol. 17, no. 1, pp. 5–17 (in Russian). DOI: https://doi.org/10.23947/1992-5980-2017-17-1-5-17

16. Sukhinov, A.A., Sukhinov, A.I. 3D Model of Diffusion-Advection-Aggregation Suspensions in Water Basins and Its Parallel Realization. Parallel Computational Fluid Dynamics, Mutidisciplinary Applications, Proceedings of Parallel CFD 2004 Conference, Las Palmas de Gran Canaria, Spain, ELSEVIER, Amsterdam-Berlin-London-New York-Tokyo, 2005, pp. 223-230. DOI: 10.1016/B978-044452024-1/50029-4. DOI: https://doi.org/10.1016/B978-044452024-1/50029-4

17. Protter, M.H., Weinberger, H.F. Maximum Principles in Differential Equation. Springer-Verlag New York, Inc. 1984, 276 p. DOI 10.1007/978-1-4612-5282-5. DOI https://doi.org/10.1007/978-1-4612-5282-5

18. Ladyzhenskaya, O.A., et al. Lineynye i kvazilineynye uravneniya parabolicheskogo tipa. [Linear and quasilinear parabolic equations.] Moscow: Nauka, 1967, 736 p. (in Russian).

19. Vladimirov, V.S., et al. Uravneniya matematicheskoy fiziki. [Equations of mathematical physics.] Moscow: Nauka, 1981, 512 p. (in Russian).

20. Tikhonova, A.N., et al. Uravneniya matematicheskoy fiziki. [Equations of mathematical physics.] Moscow: Nauka, 1977, 735 p. (in Russian).

Received 18.06 .2018
Submitted 19.06.2018
Scheduled in the issue 05.09.2018

Authors:

Sukhinov, Alexander I.,
Head of the Mathematics and Computer Sciences Department, Don State Technical University (1, Gagarin sq., Rostov-on-Don, 344000, RF), Dr.Sci. (Phys.-Math.), professor,
ORCID: http://orcid.org/0000-0002-5825-1523
Scopus Author ID: 8573972700,
WoS Researcher ID: I-1091-2016
sukhinov@gmail.com

Sidoryakina,Valentina V.,
Head of the Mathematics Department, Taganrog Chekhov Institute, Rostov State University of Economics (RINH) branch, (48, Initsiativnaya St., Taganrog, 347936, RF), Cand.Sci. (Phys.-Math.), associate professor,
ORCID: https://orcid.org/0000-0001-7744-015X
cvv9@mail.ru